**UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Subsidiary Level and GCE Advanced Level**

**MARK SCHEME for the October/November 2011 question paper**

**for the guidance of teachers**

# 9709 MATHEMATICS

**9709/31**        Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

## Mark Scheme Notes

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

• Note: B2 or A2 means that the candidate can earn 2 or 0.
        B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF     Any Equivalent Form (of answer is equally acceptable)

AG      Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD     Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO     Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO     Correct Working Only – often written by a 'fortuitous' answer

ISW     Ignore Subsequent Working

MR      Misread

PA      Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS     See Other Solution (the candidate makes a better attempt at the same question)

SR      Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

## Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1   This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

**1**  Rearrange as $e^{2x} - e^x - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent

Solve a 3-term quadratic for $e^x$ or for $u$        M1

Obtain simplified solution $e^x = 3$ or $u = 3$        A1

Obtain final answer $x = 1.10$ and no other        A1    [ ]


**2**  *EITHER*:   Use chain rule        M1

           obtain $\dfrac{dx}{dt} = 6\sin t \cos t$, or equivalent        A1

           obtain $\dfrac{dy}{dt} = -6\cos^2 t \sin t$, or equivalent        A1

           Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$        M1

           Obtain final answer $\dfrac{dy}{dx} = -\cos t$        A1

   *OR*:       Express $y$ in terms of $x$ and use chain rule        M1

           Obtain $\dfrac{dy}{dx} = k(2 - \dfrac{x}{3})^{\frac{1}{2}}$, or equivalent        A1

           Obtain $\dfrac{dy}{dx} = -(2 - \dfrac{x}{3})^{\frac{1}{2}}$, or equivalent        A1

           Express derivative in terms of $t$        M1

           Obtain final answer $\dfrac{dy}{dx} = -\cos t$        A1    [5]


**3 (i)**   *EITHER*:   Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$        M1

                Obtain quotient $x^2 + 4x + 3$        A1

                Equate remainder of form $lx$ to zero and solve for $a$, or equivalent        M1

                Obtain answer $a = 1$        A1

      *OR*:       Substitute a complex zero of $x^2 - x + 1$ in p($x$) and equate to zero        M1

                Obtain a correct equation in $a$ in any unsimplified form        A1

                Expand terms, use $i^2 = -1$ and solve for $a$        M1

                Obtain answer $a = 1$        A1    [4]

      [SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in $B$ and/or $C$, or an unknown factor $Ax^2 + Bx + 3$ and an equation in $A$ and/or $B$. The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.]

**(ii)**   State answer, e.g. $x = -3$        B1

        State answer, e.g. $x = -1$ and no others        B1    [2]


**4**  Separate variables and attempt integration of at least one side        M1

Obtain term $\ln(x + 1)$        A1

Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2,$ or $\pm \frac{1}{2}$        M1

Obtain correct term $\frac{1}{2} \ln \sin 2\theta$        A1

Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$        M1

Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2}\ln \sin 2\theta - \frac{1}{2}\ln\frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2,$ or $\pm \frac{1}{2}$)        A1√

Rearrange and obtain $x = \sqrt{(2\sin 2\theta)} - 1$, or simple equivalent        A1    [7]

**5 (i)** Make recognisable sketch of a relevant graph over the given interval
Sketch the other relevant graph and justify the given statement

**(ii)** Consider the sign of sec $x - (3 - \frac{1}{2}x^2)$ at $x = 1$ and $x = 1.4$, or equivalent     M1

Complete the argument with correct calculated values     A1     [2]

**(iii)** Convert the given equation to sec $x = 3 - \frac{1}{2}x^2$ or work *vice versa*     B1     [1]

**(iv)** Use a correct iterative formula correctly at least once     M1
Obtain final answer 1.13     A1
Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change
in the interval (1.125, 1.135)     A1     [3]
[SR: Successive evaluation of the iterative function with $x = 1, 2, \ldots$ scores M0.]

**6 (i)** State or imply $R = \sqrt{10}$     B1
Use trig formulae to find $\alpha$     M1
Obtain $\alpha = 71.57°$ with no errors seen     A1     [3]
[Do not allow radians in this part. If the only trig error is a sign error in $\cos(x - \alpha)$ give
M1A0]

**(ii)** Evaluate $\cos^{-1}(2/\sqrt{10})$ correctly to at least 1 d.p. (50.7684…°) (Allow 50.7° here)     B1√
Carry out an appropriate method to find a value of $2\theta$ in $0° < 2\theta < 180°$     M1
Obtain an answer for $\theta$ in the given range, e.g. $\theta = 61.2°$     A1
Use an appropriate method to find another value of $2\theta$ in the above range     M1
Obtain second angle, e.g. $\theta = 10.4°$, and no others in the given range     A1     [5]
[Ignore answers outside the given range.]
[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]
[SR: The use of correct trig formulae to obtain a 3-term quadratic in $\tan \theta$, $\sin 2\theta$,
$\cos 2\theta$, or $\tan 2\theta$ earns M1; then A1 for a correct quadratic, M1 for obtaining a value of $\theta$
in the given range, and A1 + A1 for the two correct answers (candidates who square must
reject the spurious roots to get the final A1).]

**7 (i)** Use a correct method to express $\overrightarrow{OP}$ in terms of $\lambda$
    Obtain the given answer                                              A~~~~

**(ii)** *EITHER*:   Use correct method to express scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OP}$, or $\overrightarrow{OB}$ and $\overrightarrow{OP}$
                  in terms of $\lambda$                                                  M1
                  Using the correct method for the moduli, divide scalar products by products of
                  moduli and express cos $AOP$ = cos $BOP$ in terms of $\lambda$, or in terms of $\lambda$ and $OP$   M1*
    *OR1*:       Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms
                  of $\lambda$                                                             M1
                  Using the correct method for the moduli, divide each expression by twice the
                  product of the relevant moduli and express cos $AOP$ = cos $BOP$ in terms of $\lambda$,
                  or $\lambda$ and $OP$                                                M1*

Obtain a correct equation in any form, e.g. $\dfrac{9+2\lambda}{3\sqrt{(9+4\lambda+12\lambda^2)}} = \dfrac{11+14\lambda}{5\sqrt{(9+4\lambda+12\lambda^2)}}$      A1

Solve for $\lambda$                                                   M1(dep*)
Obtain $\lambda = \frac{3}{8}$                                                 A1   [5]

[SR: The M1* can also be earned by equating cos $AOP$ or cos $BOP$ to a sound attempt at
cos $\frac{1}{2}$ $AOB$ and obtaining an equation in $\lambda$. The exact value of the cosine is $\sqrt{(13/15)}$,
but accept non-exact working giving a value of $\lambda$ which rounds to 0.375, provided the
spurious negative root of the quadratic in $\lambda$ is rejected.]
[SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for
$OP$ to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in such
cases.]

**(iii)** Verify the given statement correctly                                          B1   [1]

**8 (i)** Use any relevant method to determine a constant                            M1
      Obtain one of the values $A = 3$, $B = 4$, $C = 0$                        A1
      Obtain a second value                                                 A1
      Obtain the third value                                               A1   [4]

**(ii)** Integrate and obtain term $-3\ln(2-x)$                                  B1√
      Integrate and obtain term $k\ln(4+x^2)$                               M1
      Obtain term $2\ln(4+x^2)$                                         A1√
      Substitute correct limits correctly in a complete integral of the form
      $a\ln(2-x) + b\ln(4+x^2)$, $ab \neq 0$                                 M1
      Obtain given answer following full and correct working                A1   [5]

**9 (i)** Use product rule

Obtain correct derivative in any form

Equate derivative to zero and solve for $x$    M1

Obtain answer $x = e^{-\frac{1}{2}}$, or equivalent    A1

Obtain answer $y = -\frac{1}{2}e^{-1}$, or equivalent    A1   [5]

**(ii)** Attempt integration by parts reaching $kx^3 \ln x \pm k \int x^3 . \frac{1}{x} dx$    M1*

Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\, dx$, or equivalent    A1

Integrate again and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$, or equivalent    A1

Use limits $x = 1$ and $x = e$, having integrated twice    M1(dep*)

Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent    A1   [5]

[SR: An attempt reaching $ax^2 (x \ln x - x) + b \int 2x(x \ln x - x)\, dx$ scores M1. Then give the

first A1 for $I = x^2 (x \ln x - x) - 2I + \int 2x^2\, dx$, or equivalent.]

**10 (a)** *EITHER*: Square $x + iy$ and equate real and imaginary parts to 1 and $-2\sqrt{6}$ respectively    M1*

Obtain $x^2 - y^2 = 1$ and $2xy = -2\sqrt{6}$    A1

Eliminate one variable and find an equation in the other    M1(dep*)

Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalent    A1

Obtain answers $\pm (\sqrt{3} - i\sqrt{2})$    A1   [5]

*OR*: Denoting $1 - 2\sqrt{6}i$ by $R\text{cis}\theta$, state, or imply, square roots are $\pm\sqrt{R}\text{cis}(\frac{1}{2}\theta)$

and find values of $R$ and either $\cos\theta$ or $\sin\theta$ or $\tan\theta$    M1*

Obtain $\pm\sqrt{5}(\cos\frac{1}{2}\theta + i\sin\frac{1}{2}\theta)$, and $\cos\theta = \frac{1}{5}$ or $\sin\theta = -\frac{2\sqrt{6}}{5}$ or

$\tan\theta = -2\sqrt{6}$    A1

Use correct method to find an exact value of $\cos\frac{1}{2}\theta$ or $\sin\frac{1}{2}\theta$    M1(dep*)

Obtain $\cos\frac{1}{2}\theta = \pm\sqrt{\frac{3}{5}}$ and $\sin\frac{1}{2}\theta = \pm\sqrt{\frac{2}{5}}$, or equivalent    A1

Obtain answers $\pm(\sqrt{3} - i\sqrt{2})$, or equivalent    A1

[Condone omission of $\pm$ except in the final answers.]

**(b)** Show point representing 3i on a sketch of an Argand diagram    B1

Show a circle with centre at the point representing 3i and radius 2    B1√

Shade the interior of the circle    B1√

Carry out a complete method for finding the greatest value of arg $z$    M1

Obtain answer 131.8° or 2.30 (or 2.3) radians    A1   [5]

[The f.t. is on solutions where the centre is at the point representing –3i.]